

A Model for the Parton Distribution in Nuclei

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Abstract

We have extended recently proposed model of parton distributions in nucleons to the case of nucleons in nuclei.

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Recently a simple model for parton distributions in hadrons has been presented [1]. They are derived from a spherically symmetric, Gaussian distribution, the width of which reflects, via the Heisenberg uncertainty relation, the hadronic size. Two distinct parts are distinguished in a hadron: a “bare” hadron (identified with valence quarks and gluons) and hadronic fluctuations (identified with pions, which are later the source of sea partons). The parton density distributions were calculated numerically using a Monte Carlo technique, and good agreement with deep inelastic scattering data was reported [1].

In this paper we have used the same method to calculate parton distributions in nuclei. Preserving the simplicity of the model and using the standard nuclear structure we were able to successfully describe the $F_2(x)$ in nucleus over the whole range of x . The only changes introduced were dictated by the presence of other hadrons around the one under investigation in a similar fashion as in our previous work on this subject [2]. However, in the present case we were able to describe also the region of small x , which shows effects of shadowing and therefore is of particular interest to the physics of heavy ion collisions [3].

Let us first recollect the main features of the model [1], which we shall follow as close as possible (in particular the choice of kinematics and notation will be the same). Hadron is visualised there as a composition of *bare hadron* and *hadronic fluctuations*. The former are made out of valence quarks and gluons and the latter are the source of sea quarks and gluons. They are formed mainly by the pion. One starts with the hadron at rest in which frame all partons are supposed to be distributed according to a spherically symmetric, Gaussian distributions. Such a form is natural because of a large number of interactions binding partons in the hadron. The only parameter here is the width of this distribution, the value of which is expected to be of the order of a few hundred MeV (both from the Heisenberg uncertainty relation applied to the hadron size and from the primordial transverse momentum of partons observed in deep inelastic collisions). This parameter encompasses also all perturbative QCD effects present due to the initial state emission and therefore depends on the scale Q_0^2 ¹. The goal of this approach is not so much the full wave function of the hadron, as the probability of finding a single parton with the four momentum k probed by external current with four-momentum q . Therefore all other partons are treated collectively as a single remnant with the four momentum r . Because the

¹Actually, when confronting flavour dependent data the number of parameters is enlarged because each quark flavour can have its own width; the same is true for gluons, cf. [1]. In such situation the percentages of each species in the momentum sum rule is also a kind of parameter.

above prescription provides us only with the three-momentum of the probed parton it is assumed that the energy component is equal to the parton (current) mass plus a Gaussian variation with the same width as above. It means that the parton can be off-shell at the scale Q_0^2 and fluctuates with a life-time given by the nucleon radius.

The reaction takes place in a coordinate system in which the negative z -direction points along the current which probes the hadron. One uses the light cone momentum fraction x of the parton defined as $k_+ = xp_+$ (where p is the four-momentum of the hadron). The final four-momentum of the scattered parton denoted by j must satisfy the following condition: $0 < j^2 < W^2$ (where W is invariant mass of the hadronic system). This is equivalent to $0 < x < 1$. When masses of quarks are neglected the same condition must be satisfied by the four-momentum r of the remnant: $0 < r^2 < W^2$. The parton distributions are then calculated by a Monte Carlo code ². The momentum k component of the parton to be probed by current with virtuality Q_0^2 and four-momentum q is chosen from the Gaussian distribution described before. Actually the values of Q_0^2 is a free parameter expected to be of the order of 1 GeV². This makes it possible to calculate the four-momenta j and r . Events are accepted if the exact kinematical constraints mentioned above are fulfilled. In this way one obtains as a result the valence parton distribution $f_v(x; Q_0^2)$ (“bare” hadron) ³. The sea parton distribution is given by the pionic component of the nucleon: $f_s(x; Q_0^2) = \int \frac{dy}{y} f_\pi(y, Q_0^2) f_{pion}(x/y; Q_0^2)$. It is the convolution of the pion distribution function in the nucleon, $f_\pi(x; Q_0^2)$, and the parton structure of pion $f_{pion}(x; Q_0^2)$. This in turn is obtained from the same Gaussian primordial distribution as used for valence partons. The characteristic behaviour of the sea partons is then derived from the pion distribution in the nucleon, which was again parametrized

²In our work we have used a simplified version of model [1] with the same distributions for all valence quarks and without evolution in Q^2 . We shall also not address gluon distributions, which in [1] were assumed to have the same basic Gaussian shape as the valence quarks. The presence of gluons will be accounted for only in the momentum sum rule, where part of momentum will always be allocated to the gluonic component.

³One has to remember, however, that it is, the so called, Bjorken variable $x = x_{Bj} = \frac{Q^2}{2p \cdot q}$, which is accessible experimentally whereas [1] starts from the light cone target rest frame variable $x = x_{LC} = \frac{k^+}{p^+}$ (where $p = (M, 0, 0, 0)$) with a fixed resolution $Q^2 = Q_0^2$. Whenever one is interested in some higher Q^2 , the QCD evolution in the initial state must be performed. In this case the $x = x_{Bj}$ picked out of the proton at Q^2 will be smaller than the $x_{LC} = k^+/p^+$ one has started with at Q_0^2 . The remaining energy is radiated into the final state as jets. If the invariant mass squared of all jets in the final state is M_X^2 , then $x_{LC} = x_{Bj} \left(1 + \frac{M_X^2}{Q^2}\right)$. In this case, if one only scatters a quark with no further radiation, then $x_{LC} = x_{Bj} \left(1 + \frac{m_q^2}{Q^2}\right) \simeq x_{Bj}$. This clarifies the limits of applicability of the model used here.

by Gaussian distribution with a smaller width equal to 0.052 GeV reflecting the smallness of the pion mass. The total nucleon structure function is then equal to: $F(x; Q_0^2) = f_v(x, Q_0^2) + f_s(x; Q_0^2)$.

We shall now apply this model to deep inelastic collisions with nuclei, $l + A \rightarrow l' + X$. As usual in such cases our aim will be the nuclear structure function $F_2^A(x_A)$, which shows a characteristic pattern: shadowing at small x , followed by antishadowing at $x \sim 0.1 \div 0.3$, followed in turn by a very pronounced deep around $x \simeq 0.7$ and a kind of cumulative effect for $x \rightarrow 1$. This subject has already long history and a vast literature (cf. [4] for review). Our aim is to see if, and under what conditions, the model proposed in [1] for free nucleons can be also applied to nuclei. In the approach presented here, from many possibilities mentioned in [4], we have selected the picture in which collision proceeds on nucleons bounded in nuclei, looking for nuclear partonic distributions as a sum of distributions of bound nucleons. The corresponding nuclear structure function can be then written as simple convolution of nuclear and nucleonic components:

$$\frac{1}{x_A} F_2^A(x_A) = A \int dy_A \int \frac{dx}{x} \delta(x_A - y_A x) \rho^A(y_A) F_2^N(x). \quad (1)$$

Here $F_2^N(x)$ denotes a nucleon structure function inside the nucleus and $\rho^A(y_A)$ is the nucleon distribution function in the nucleus. Variables x_A/A and y_A/A are the corresponding Björken variables for the quarks and nucleons in the nucleus, respectively (with nucleonic and nuclear longitudinal momenta given by p^+ and P_A^+). The presence of nucleus is summarised here by the nucleon distribution function ρ and the simplest approach one could think of would be to use available information on it and keep the nucleonic structure functions unchanged. For example, in the framework of the relativistic mean field theory (RFM) [6] leptons interact with nucleons with density ρ^A , which are moving in a constant average scalar U_S and vector U_V potentials (defined in the rest frame of the nucleus as $U_S = -400 \frac{\rho}{\rho_0}$ MeV and $U_V = 300 \frac{\rho}{\rho_0}$ MeV, in both cases $\rho_0 = 0.17 \text{ fm}^{-3}$). As a result (cf. [7]), in the Relativistic Fermi Gas approximation the following form of the nuclear density function is obtained:

$$\rho^A(y_A) = \frac{4}{\rho} \int \frac{d^4 p}{(2\pi)^4} S_N(p^0, \mathbf{p}) \left[1 + \frac{p_z}{E^*(p)} \right] \delta \left(y - \frac{p_0 + p_z}{\mu} \right). \quad (2)$$

The ρ is nuclear density (which is different from the density of infinite nuclear matter ρ_0); $S_N = n(p) \delta \{p^0 - [E^*(p) + U_V]\}$ denotes nucleon spectral function with $n(p)$ being Fermi distribution of nucleon momenta inside the nucleus, $p \in (0, p_F)$. The corresponding Fermi momentum $p_F = (3/2\rho\pi^2)^{1/3}$. The nucleon chemical potential

$\mu = M_A/A$ (which in RMF can be shown to be $\mu = E^*(p_F) + U_V$) and $E^*(p) = \sqrt{p^2 + m^{*2}}$ with $m^* = m - U_S$. After integration (2) simplifies to

$$\rho^A(y_A) = \frac{3}{4} \frac{[v_A^2 - (y_A - 1)^2]}{v_A^3}. \quad (3)$$

Here $v_A = p_F/E_F^*$ and y_A takes the values given by the inequality: $0 < (E_F^* - p_F) < \mu y_A < (E_F^* + p_F)$. In this way the motion of the nucleon inside the nucleus is parametrized here by two parameters: nuclear density ρ which determines Fermi momentum p_F and nucleon chemical potential μ . Out of these two, the nucleon chemical potential is essentially constant (and equal to $\mu = 8$ MeV), except for a few very light nuclei. The nuclear density ρ due to the finite size of the nucleus can vary from $\rho \simeq 0.1 \text{ fm}^{-3}$ for light nuclei to $\rho \simeq 0.17 \text{ fm}^{-3}$ for heavy nuclei and nuclear matter. Taking $\rho \simeq 0.12 \text{ fm}^{-3}$ for the average density of nucleons in ^{56}Fe with $p_F = 240 \text{ MeV}$ and using (3) results in dashed curve in Fig. 1, which is completely off data. This means that RFM approach to deep inelastic scattering on nuclei fails, at least in the form presented here. One could play with nuclear parameters, for example by increasing chemical potential μ to make the structure of deep around $x \sim 0.5 \div 0.7$ more pronounced and more similar to the experimental data. This would, however, be difficult to justify and the shadowing/antishadowing structure seen in data at smaller x would remain unexplained. There are then two possibilities: either to keep nucleon structure functions unchanged and try to change ρ by including correlations [11] and/or cluster structure of nucleons inside the nucleus [12] or to decide to change the nucleonic structure functions. Because the first approach will essentially not affect the shadowing (or even antishadowing) region of x , we have opted for the second possibility. In what follows we shall therefore use, for nuclear structure function, the nuclear density ρ^A as given by eq. (2) with $\mu = 8$ MeV and with $U_S = U_V = 0$ (i.e., $m^* = m$). This is to avoid the possible double counting of nuclear interactions, which in our model can be expressed either by potentials $U_{V,S}$ (treated now as free parameters) or by changes of the free nucleon structure functions discussed below. Because we have decided (as said above) for the latter possibility we have consistently set values of potentials equal to zero. It means that we are considering here a model of Fermi gas with modified nucleons.

Our Monte Carlo calculation for collision on nuclei has been performed in the same way as in [1]. The only difference was that now the nucleon three-momentum is not fixed but chosen from distribution $\rho^A(y_A)$ as given by eq. (2), i.e., we account for the Fermi motion of nucleons in nucleus. This introduces a kinematical medium effect produced by Lorentz transformation of the parton momenta from the nucleonic

to the nuclear rest frame. This Fermi motion of nucleons also affects invariant mass W , four momentum $r = p - j$ and Bjorken variable $x = k_+/p_+$ and in this way influences the calculated structure function $F_2^B(x, p)$ of the bound nucleon making it momentum dependent. Analytically it would mean that one extends our simple convolution formula (1) replacing $F_2^N(x)$ by momentum dependent $F_2^B(x, p)$ and integrates over momentum p , i.e., instead of (1) one uses formula

$$F_2^A(x_A) = \frac{4A}{\rho} \int dy_A \int \frac{d^4p}{(2\pi)^4} S_N(p) \left[1 + \frac{p_z}{E^*(p)} \right] \delta \left(y_A - \frac{p_+}{\mu} \right) F_2^B \left(\frac{x_A}{y_A}, p \right). \quad (4)$$

The other modifications mentioned before must be made by some suitable changes in parameters of the original nucleonic structure functions. Because we are not differentiating here between partons of different flavour there are only three such parameters, which can be argued to be affected by nuclear medium: the widths of the original gaussians (i.e., transverse primordial distributions), the same and equal $\sigma_q = 0.18$ GeV for both valence quarks (“bare” hadron) and for nucleonic pions (sea quarks) and the width of the pionic distribution in nucleon equal $\sigma_\pi = 0.052$ GeV. It turns out that, in order to get reasonable description of data, it is enough to modify only σ_q (keeping it again the same for both types of quarks) by decreasing it to the value $\sigma_q = 0.165$ GeV and centering pionic energy distribution not at the pion mass $m_\pi = 0.14$ GeV but at $m_\pi = 0$. This can be seen in Fig. 1 where we present results of calculations of ratio $R(x) = f_2^A(x)/f_2^D(x)$ for ^{56}Fe (performed using average value of nuclear density $\rho = 0.12 \text{ fm}^{-3}$, $p_F = 240 \text{ MeV}$). The mentioned results are presented by open squares. Such a change in the width of initial partonic distribution can be naturally explained by the expected swelling of nucleons in nuclear matter [2, 4]. Smaller spread of momenta caused by the internucleonic interactions corresponds, due to the uncertainty relation, to bigger spread in coordinate space (the “vacuum” in nucleus is obviously different from that outside the nucleus). Although the idea is similar to what was proposed a long time ago in [2] the effect of swelling is this time modelled not by changing the bound nucleon mass but by changing the intrinsic motion of partons inside such nucleon. What concerns the change made in the center of the energy distribution of nucleonic pions one can argue, following for example [13], that in nuclear matter such pions, interacting strongly with nuclear matter and coupled to the nuclear collective modes, seem to have indeed vanishing effective mass. In any case such change is essential in getting agreement with data. With only swelled nucleon and without decreasing m_π (understood as a position of maximum in the energy distribution of nucleonic pions) results in curve denoted by open diamonds in Fig. 1. The other parameters were left unchanged (in particular, the fraction of momentum carried by the sea quarks is left 7.7% as in [1]).

Contrary to what has been presented in [2] this time we have also reproduced (at least partially) the shadowing/antishadowing region of x . It has to be admitted here, however, that there is a price in obtaining these results (see open squares in Fig. 1). The momentum sum rule is now violated by $\sim 2\%$, which in our case means that this fraction of nucleonic momenta is taken by gluons (in addition to what they already had in free nucleons). If we strictly impose the momentum sum rule (i.e., if we assume that there is no additional transfer of the momenta to gluons), it results in changing the momentum carried by quarks to 9.7% and spoils the agreement with data in the vicinity of the antishadowing region (cf. Fig. 1, open triangles).

To conclude, we have demonstrated that the simple model of partonic distributions in nucleons developed recently in Ref. [1] can also successfully describe partonic distributions in nuclei by: (i) using standard description of nucleonic Fermi motion, (ii) changing by small amount (about 10%) the Gaussian widths of the initial partonic components and (iii) centering the energy distribution of nucleonic pions on $m_\pi = 0$ instead 0.14 GeV. These changes correspond to the valence quark distributions in nuclei being shifted towards the lower values of x and the sea quark distribution being relatively spread out and shifted towards the higher values of x . Also nucleons in nucleus allocate a bit more momentum in the gluonic component. Altogether, by these simple means a good agreement with data was obtained for the whole range of x except of really small values of $x \leq 0.06$ where such small changes are definitely not sufficient and convolution model is hardly justifiable [4]. We consider it remarkable that by changing only one parameter describing the primordial diffusiveness of partons (in momentum space) in the free nucleon and shifting pion distribution in way suggested by nuclear matter calculations we were able to describe experimental data in a fairly reasonable way in a very broad region of x . Actually, our results are quite robust to changes in initial values of parameters describing partonic distributions in free nucleon (for example, a 20% change in the quark distribution in pion does not change results presented in Fig. 1). Our analysis is obviously simplified, as we have not differentiated between quark flavours or accounted for the presence of gluons only via the momentum sum rule. Also our convolution approach, cf. eq. (1) is not suitable for small values of x . But already at this level it shows that model [1] is highly suitable for application to nuclear partonic distributions because of simplicity and cogency of the parametrization chosen there.

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Figure Captions:

Fig. 1 Results for $R(x) = F_2^A(x)/F_2^D(x)$ for ^{56}Fe . Data are from [8] - full diamonds and from [9, 10] - full circles. See text for explanations.

